

SEÇÃO: VARIA

THE LOGIC OF FICTION: A PARAconsistent AND PARacomplete APPROACH

A Lógica da Ficção: uma abordagem paraconsistente e paracompleta

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Resumo: Nesse artigo, defendemos e desenvolvemos a ideia segundo a qual o discurso ficcional é melhor descrito em termos de uma lógica paraconsistente e paracompleta. Usando uma Lógica de Evidência e Verdade como ponto de partida, LET_K , apresentamos uma semântica bivalorada não-determinística que lida com a verdade ficcional e metaficcional ao introduzir o operador $\circ$. Em seguida, lidamos com algumas objeções, e apresentamos a lógica LET_K^+ como uma maneira intuitiva de apresentar diferentes noções de verdade na ficção.

Palavras-chave: Lógica da ficção. Lógica paraconsistente. Lógica paracompleta. Lógicas de Evidência e Verdade.

In this paper, we defend and develop the idea that fictional discourse is best described in terms of paraconsistent and paracomplete logics. Using a Logic of Evidence and Truth as a starting point, LET_K , we present a nondeterministic bivaluation semantics that deals with fictional and metafictional truth by introducing the $\circ$ -operator. After that, we deal with some objections, and we present the logic LET_K^+ as an intuitive alternative of presenting different notions of truth in fiction.

Key words: Logic of fiction. Paraconsistent logic. Paracomplete logic. Logics of Evidence and Truth.

1 Introduction

From a standard position on truth, determining the truth-value of ordinary propositions is straightforward. For example, if a football player kicks a ball, the sentence “the football player has kicked a ball” expresses a true proposition. This might occur, in our rough view, because it is a fact that someone, who is a football player, kicked a ball. However,

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independent of our position about the concept of truth, how does this concept apply when dealing with fictional discourse? When we say that it is true that Spider-Man was bitten by the species *Araneus Ocorpeus*, what do we mean by that? In Spider-Man's stories, within the narrative, this fictional species of spider (which is a fictional species) bites Peter Parker and then he gains his powers. Nevertheless, Peter Parker is not a flesh-and-blood man, nor is this specific spider the kind of creature we see in our daily lives. So, in some sense, we could not say that he was really bitten by a spider. Neither the spider nor Peter Parker are the kind of objects that participate in any concrete causal relation. It seems that we can, for example, say that it is true that black widows are deadly spiders or that they have red marks on their backs. In this case, we are talking about an actual type of spider species; one that we can encounter in some locations of the world. Still, despite these remarks, we typically regard the statement 'A radioactive spider bit Peter Parker' as fictionally true.

But more unusual than these cases is when we talk about truth inside fiction. Suppose that in one volume of Spider-Man we have the following dialogue:

(Mary Jane) "Peter, be honest with me, are you Spiderman?"

(Peter Parker) "No, I'm just Peter."

In this case, would we say that Peter is lying to Mary Jane? If so, could we say that the sentence said by Peter expresses a falsity? If it isn't false, would we say that it expresses the truth? As we see in literature, the answer to these questions was shortened to something like: well, the fictional discourse does not deal with matters of fact, which means that it is wrong to take sentences internal to fiction as facing any value. We aim to demonstrate that it is reasonable to speak of truth (and falsity) internal to fictional narratives. Additionally, we can show how it is related to truth and falsity in other discourses about fiction.

Given this, we want to propose that the proposition expressed by sentences like "A radioactive spider bit Peter Parker" is fictionally true. We will argue that fictional truth is fundamentally different from truth simpliciter, even at the propositional level. Whereas truth is always classical, fictional discourse allows for overlaps and undeterminedness, that is, fictional discourse is paraconsistent and paracomplete. Starting with a few examples motivating this divergence between classical and fictional truth, we will see how we can initially characterize fictional truth in terms of a family of logics called "Logics of Evidence and

Truth” (LETs), explored by Carnielli and Rodrigues (2021), Carnielli and Rodrigues (2015), Carnielli and Rodrigues (2019), Coniglio and Rodrigues (2024). The suggestion of pursuing non-deterministic semantics to obtain parafinite logics that deal with fictional discourse is due to Peron and Antunes (2022). In Section 5, we will compare our approach with theirs. This is a starting point to think of fictional discourse in terms of LETs, and our choice to work with LET_K is for reasons of simplicity: it is a minimal LET. Later on, we suggest other LETs that are stronger that can also be considered to represent fictional discourse.

2 Examples of Fictional Assertions

Before clarifying what we mean by fictional truth, we should distinguish the levels of discourse that fiction presents to us. First, intuitively, there are at least two levels of discourse: we have the discourse as presented in fiction, that is, internal to the work; and we have the discourse that we say about fiction, that is, external to fiction. For example, when Sherlock Holmes says ‘Nice to meet you, Mr. Watson,’ it is a representation of an utterance made by a fictional character inside the fictional narrative. Contrast this with some speculation about the story, for instance, if we say that “Sherlock Holmes seems the kind of character who compliments no one other than Mr. Watson”. This is not a sentence presented in the original narrative. In this last case, we treat Sherlock Holmes not as a flesh-and-blood detective, but as a character inside a story. In fact, this is an utterance made from a point of view external to the fictional narrative. Recanati (2018) was the author who dealt with these issues contemporarily. In his article, “Fictional, Metafictional, Parafictional” he elaborated a division on three kinds of sentences. According to him, we could elaborate sentences that refer to the fictional characters as flesh-and-blood persons. That is the kind of sentence we find inside a fictional narrative.³ On the other hand, we could elaborate sentences that refer to fictional characters as fictional characters. This last one is the kind of sentences we elaborate on outside of a fictional narrative. Both are called, respectively, fictional and metafictional sentences. However, as Recanati has noticed, we could formulate one more kind of sentence: the parafictional. We call a sentence parafictional when it is preceded (implicitly or explicitly) by the expression “according to the story...”. For example, “According to *A Study in Scarlet*,

³ Some philosophers would say that what characterizes these sentences is the pretense feature. But this is not a debate we want to raise here.

Sherlock Holmes is a clever detective”. Notice that here we are not quoting some utterance presented in the story *A Study in Scarlet*, even though we are talking about what is (or is not) the case in the narrative.

Summing up, when engaging with fiction, we encounter three distinct levels of discourse: fictional, parafictional, and metafictional. To say what is true or not at the metafictional level seems less doubtful. For instance, cases such as the sentence “Bilbo was a character created by John R. R. Tolkien” are more akin to what we are used to dealing with in our daily lives. The proposition expressed is factual, and it is more straightforward to evaluate whether the proposition expressed is true or false (in this case it is a true proposition). The same could not be said about the fictional and parafictional discourses.

In the case of fictional discourse, most of the authors would say that it is not truth evaluable, even though fictional sentences do raise other philosophical issues concerning fictional terms. Kendal Walton (1990), for example, would defend that in fictional discourse we play a series of imaginative games and pretenses in order to represent objects, characters, and fictional narratives. Since in this level of discourse we are just pretending to assert, this is not something we should take seriously. So, it makes no sense to question the truthfulness of fictional propositions. Based on such motivations, the most common approach is to say that there is no notion of truth associated with fiction.⁴ but the parafictional discourse is more debatable. With this discourse, we say something about the fiction (which would be an external point of view), but its truthfulness is determined by what is said in the fictional narrative. As Recanati (2018, p. 5) said: “Parafictional utterances talk about the story, like metafictional utterances, but they do so from a perspective internal to the story, by describing the world as it is in the story”. In that case, some philosophers can interpret the parafictional discourse as being a type of metafictional discourse. Nevertheless, in the end, most of what we say is true parafictionally depends on what is said inside the fictional narrative.

Hence, in some sense, we should not abandon the notion of truth inside Fictional discourse. As we see in Miranda (2013), there are even those who argue that we could have an argumentative aspect in fiction. Because of that, we want to propose a more precise view of how to understand the notion of truth within fictional discourses. Instead of putting off the debate about the truth in fiction, we will formulate a more precise notion of fictional truth.

⁴ We could even say some find it absurd to question about the truth in fictional contexts.

Moreover, we should differentiate the truth we have in fiction and the truth we use in other contexts (e.g., in science), because, different from factual contexts, in fiction, the author does not have any obligation to, e.g., be consistent. We now turn to the aspects of fiction we aim to address.

2.1 Fictional Truth

Roughly, fictional truth is what is true in a certain narrative.⁵ Even if it is not considered true as a fact, some things could be called true inside a fictional narrative. For example, inside the fictional universe created by Joe Shuster and Jerry Siegel, it is true that Superman's weakness is something called Kryptonite. But, we ought to notice, different from what we see, e.g., in biology, this is true not because there is a flesh-and-blood man (or that there is a Kryptonian) with superpowers in our actual world. This is true because the authors created the Superman fictional universe in such a way that "Superman's weakness is Kryptonite" expresses a fact in that universe. In other words, the proposition is fictionally true.

2.2 Fictional Falsehood

Conversely, taking the example exposed in the Introduction, if Peter has said "No, I'm not Spider-Man", let's think about what we, spectators of such work, were invited to reflect. The intuitive thought is that, inside the narrative presented by Steve Ditko and Stan Lee, the character has expressed a false proposition. This fictional universe is created in such a way that it is a fact that Peter Parker is the person behind the superhero costume. So it is true that Peter Parker is the Spider-Man. Denying this fact, which is just what Peter did, is lying (because Peter Parker knows he is Spider-Man). In other words, the sentence "No, I'm not Spider-Man" expresses a fictional falsehood when asserted by Peter Parker in the world of Spider-Man. It becomes even more noticeable in this case how the truth-value of a sentence depends on the fictional universe, or fictional world, we are considering. Let's suppose Peter Parker asserts "I am the only Spider-Man". If we are considering the Tobey Maguire Spider-Man depicted in the 2002 trilogy, this is a true sentence. However, if we are considering the Spider-Man movie from

⁵ Here we do not want to present a way to uncover these truths, which would demand a theory of interpretation. We aim to identify what may count as Fictional Truth, analyze it, and refine the concept

2021, starring Tom Holland, Andrew Garfield and Tobey Maguire, the sentence “I am the only Spider-Man”, asserted by the Tobey Maguire version of Spider-Man, is not true anymore.

3.3 Fictional Overdeterminedness

Intentionally or not, sometimes the authors create a fictional universe that engenders a contradiction (a universe that accepts A and not-A). This is what we call fictional overdeterminedness. Three cases of occurrence of contradictions in stories are identifiable: (i) in serialized comics, where fictional worlds are merged to the point where we have a contradiction in this fictional world. For example, in some volumes of Thor’s stories it is said that he is the only one who could raise Mjörnir (the distinctive hammer of the character in the comics), but, in Avengers (a comic series that gathers some heroes, including Thor) we see Captain America raising Mjörnir. In this case, the combinations of these fictional universes produce contradictions. These types of contradictions we call parafictional contradictions;⁶ (ii) one other case can be exemplified by the movie *The Dark Knight*, in which the character Joker tells different people different stories about how he got his scars. He states that his father cut him, then contradicts it by claiming that he did it himself for his wife. This contradiction is likely intentional, perhaps used to evoke uncertainty in the spectator, or even to show that the Joker is not a reliable character. But the point here is that we could do those things in fictional narratives and, then, our attempt to make the notion of fictional truth more precise must take this aspect into account; These types of contradictions are produced by lying or faulty characters;⁷ (iii) a third use of contradictions in fictional stories is a more substantial one, in which there is a contradiction within one fictional story that is true within that world. These types of contradictions are fictional contradictions. A paradigmatic case of that is the box that is empty and occupied in *Sylvan’s Box* by Priest (1997). In that story, Graham Priest is going through Richard Sylvan’s works after his passing away when he discovers a box labeled “Impossible Object”. Upon opening the box, he finds it empty and occupied at the same time. The story centers around the fact that this contradiction is true within this fictional world. Paradoxical situations can also serve positively to the narrative to produce certain literary

⁶ We thank one reviewer for the suggestion of this name.

⁷ Not only characters, but narrators can also be considered unreliable, in which case they can also produce a contradiction.

effects as pointed by Egamnazarova (2023). The case we are really interested in characterizing here is (iii), because that would be the case of a true contradiction.

4.4 Fictional Undeterminedness

Finally, we are aware that authors often deliberately leave gaps in the fictional world to leave the interpretation open to the reader. Sometimes, though, those gaps are not deliberate. But, in the end, in works of fiction, we have cases where it is unsettled whether *A* or not-*A* is the case. For example, in the literature of Conan Doyle, we cannot determine whether Sherlock Holmes has exactly 90,000 hairs on his head. So, in this fictional universe, either Holmes has 90,000 hairs on his head or he does not. Moreover, there are also cases where such indeterminacy is deliberately employed to give the reader some interpretative freedom. For instance, by the end of the movie *Blade Runner*, it is never confirmed to the audience whether Deckard is a replicant or not. This is done to make the spectators reflect upon what is the case in this fictional world. Yet, again, this is a tool often used by the authors, so, our notion of fictional truth should deal with this peculiarity. Another good example of a purposeful undeterminedness occurs in *Dom Casmurro*, a romance written by Machado de Assis. In that story, one of the central questions is whether Capitu has cheated on Bentinho, the two main characters. This question is left open for the readers to discuss, and it is this undeterminedness that has caused great debates among literary students and researchers of this subject, as pointed by Miranda (2013).⁸

One objection to fictional indeterminacy is the following one: some things are left unspoken but are presupposed; for example, that Sherlock Holmes has a pair of eyes, one mouth, etc. For the rest of the things that are not presupposed, we can simply think of them as fictionally false. Therefore, it would be fictionally false that “Sherlock Holmes likes brazilian music”, for example. The problem here is that this leaves little room for fictional worlds to incorporate new information. For example, suppose Conan Doyle, the author of *Sherlock Holmes*, has (probably) never thought about Holmes’ musical taste and his acquaintance with brazilian music. In our view, we would say that “Sherlock Holmes likes brazilian music” is

⁸ Even though there is evidence, in the romance, that Capitu has cheated on Bentinho, and also there is evidence that Bentinho is paranoid about something that did not happen, the question remains open. That is, fictional truth would not settle for evidence; it needs an assertion of a proposition or its negation. In that sense, our fictional reading of Logics of Evidence and Truth is different from the standard interpretation in terms of evidence.

undetermined. However, our opponent would say that his sentence is fictionally false. What if Conan Doyle wanted to add a story in which Sherlock Holmes was fond of brazilian music? In our case, it's only a matter of adding information. In our opponent's view, the proposition should change from fictionally false to fictionally true, which is not as intuitive as going from undetermined to true.

There are two ways in which "Sherlock Holmes likes brazilian music" is undetermined. In one sense, it says that "In the story *X*, the proposition 'Sherlock Holmes likes brazilian music' is undetermined." In a second case, it says that "In the fictional world of Sherlock Holmes, 'Sherlock Holmes likes brazilian music' is undetermined." Our approach always refers to a fictional background Γ which is the set of texts considered to compose the fictional world. If we restrict Γ to one single text such as *A Study in Scarlet*, the undeterminedness will be relative to such text. It is possible, however, to consider the "universe" of Sherlock Holmes created by Conan Doyle. In that case, the fictional background would be $\Gamma_1 \cup \Gamma_2 \cup \dots \cup \Gamma_n$, in which each Γ_i , such that $1 \leq i \leq n$, is a text in the works of Conan Doyle.

3 Fictional truth is non-classical

The examples above illustrate that fictional truth differs in two key respects from the classical notion of truth: a fictional fact is about a fictional world created by someone. As such, it can have contradictions or undeterminedness, but it does not need to. Of course, a narrative is always incomplete in the sense that it will not describe every fact of the fictional world, unless the story begins with a claim that the fictional world is exactly like ours.⁹

Therefore, when looking for a logic to represent fictional truth, we want a negation weaker than the classical one: we want a negation such that the Principle of Explosion (PE) and the Principle of the Excluded Middle (PEM) are not valid. In the classical scenario, A and $\neg A$ cannot hold together, and at least one of A or $\neg A$ must always hold.

⁹ We thank one of the reviewers for suggesting this observation.

$$PE \frac{A \quad \neg A}{B}$$

$$PEM \frac{}{A \vee \neg A}$$

Imposing these two conditions on fictional narratives is overly restrictive, if not impossible to achieve: first, it would require that a story be completely consistent in the sense that no contradictions are implied by the story; otherwise, it would explode the fictional world. On the other hand, it would require that everything about the fictional world be determined, that is, it would require that everything about the fictional world be asserted or implied from the assertions of the story, which would, at the very best, make the storytelling a boring act of listing facts and defy the idea that fictions are exercises of imagination. Nevertheless, it is desirable that fictional discourse preserves most of our intuitions about deductive reasoning: for example, it should validate detachment of the material conditional, or the law of adjunction. It is important to stress that we are proposing a logic to represent deductive reasoning in fiction. In that sense, we do not exhaust all cases of inference in works of fiction: we are only pointing out the minimal inferences that are possible to make. In a first moment, we will consider only fictional discourse by introducing LET_K^- . In LET_K , an extension of LET_K^- , we can consider mixed contexts in which metafictionality also plays a role.

4 Logics of Evidence and Truth

4.1 LET_K^- : an initial characterization of truth

Before contrasting truth simpliciter and fictional truth, we must first examine the latter independently. That is, we need a paraconsistent and paracomplete logic that still preserves most properties about truth. In this subsection, we consider the logic LET_K^- , which is the fragment of LET_K without the consistency operator. LET_K was presented in Coniglio and Rodrigues (2024).

The following definition is adequate for our initial purposes of characterizing fictional reasoning, which we'll discuss in the next section and expand to LET_K :

Definition 1. For all Γ_k such that Γ_k is a set of fictional assertions of k , k is a fictional world, for all formulae φ ,

$$\Gamma_k \vdash_{LET_K^-} \varphi \Leftrightarrow \Gamma_k \text{ fictionally implies } \varphi \text{ in the story } k$$

One important remark concerns what is included by Γ_k . It is a set of assertions of the fictional world k . This fictional world may be determined in various ways: it may consider not only one text, but multiple narratives that are considered to be part of the same universe. Either way, this choice is not logical. It is an interpretative task to determine what the basic set of fictional assertions of a narrative is. Furthermore, the set of assertions of k may not include only fictional assertions about it, but also metafictional and parafictional assertions.

LET_K^- , presented in Coniglio and Rodrigues (2024, p. 565), can express that:

1. A holds and $\neg A$ doesn't hold
2. A doesn't hold and $\neg A$ holds
3. Both A and $\neg A$ hold
4. Neither A nor $\neg A$ hold

In the original interpretation of LETs as logics of evidence, “ A holds” is interpreted as “There is evidence for A ”, and “ A doesn't hold” is interpreted as “There is no evidence for A ”. As discussed in Barrio (2018), it is controversial whether LETs require an interpretation in terms of evidence, because it also allows a dialethic interpretation. Here, we will not be concerned with the question whether LETs have a canonical interpretation, which is better discussed in the paper by Barrio. However, what we intend to show is that LET_K^- and its extensions also allow for a fictional interpretation, which is not rejected by Carnielli and Rodrigues nor Barrio. In our present framework, we are concerned not with evidence, but with fiction: “ A holds in k ” is interpreted as “ A is true in the fictional world k ”, and “ A doesn't hold in k ” is interpreted as “ A is not true in the fictional world k ”. Thus, we identify four possible states relative to a fictional world k . LET_K starts by denying PE and PEM, which, if valid, would make it impossible to consider the scenarios 3 and 4, leaving only those rules for positive classical logic (PCL). Like Carnielli and Rodrigues (2015), we will heuristically check if the rules of LET_K preserve fictionality (in the original case, it preserves evidence).

$$\wedge I \frac{A \quad B}{A \wedge B}$$

$$\vee I \frac{A}{A \vee B}$$

$$\vee I \frac{B}{A \vee B}$$

$$\rightarrow I \frac{[A] \quad B}{A \rightarrow B}$$

Adding elimination rules, we end up with positive intuitionistic logic (PIL):

$$\wedge E \frac{A \wedge B}{A}$$

$$\wedge E \frac{A \wedge B}{B}$$

$$\vee E \frac{A \vee B \quad \begin{array}{c} [A] \\ C \end{array} \quad \begin{array}{c} [B] \\ C \end{array}}{C}$$

$$\rightarrow E \frac{A \rightarrow B \quad A}{B}$$

Finally, we add the axiom of extended excluded middle for the implication, obtaining positive classical logic (PCL):

$$\text{ly} \quad \text{CL} \rightarrow \overline{A \vee (A \rightarrow B)}$$

The rules of LET_K^- for negation associated with other connectives mirror the rules specified above:

$$\neg \wedge I \frac{\neg A}{\neg(A \wedge B)}$$

$$\neg \wedge I \frac{\neg B}{\neg(A \wedge B)}$$

$$\neg \vee I \frac{\neg A \quad \neg B}{\neg(A \vee B)}$$

$$\neg \rightarrow I \frac{A \quad \neg B}{\neg(A \rightarrow B)}$$

Also, for the case of elimination rules:

$$\neg \vee E \frac{\neg(A \vee B)}{\neg A}$$

$$\neg \vee E \frac{\neg(A \vee B)}{\neg B}$$

$$\neg \rightarrow E \frac{\neg(A \rightarrow B)}{A}$$

$$\neg \rightarrow E \frac{\neg(A \rightarrow B)}{\neg B}$$

$$\neg\wedge E \quad \frac{\neg(A \wedge B) \quad \frac{[\neg A] \quad \dots}{C} \quad \frac{[\neg B] \quad \dots}{C}}{C}$$

Finally, the rules for double negation:

$$\neg\neg I \quad \frac{A}{\neg\neg A} \qquad \neg\neg E \quad \frac{\neg\neg A}{A}$$

LET_K 's notion of logical consequence, presented for BLE in Carnielli and Rodrigues (2015, p. 19) has the following properties:

1. Reflexivity: if $A \in \Gamma$, then $\Gamma \vdash A$
2. Monotonicity: if $\Gamma \vdash B$, then $\Gamma, A \vdash B$, for any A
3. Cut: if $\Delta \vdash A$ and $\Gamma, A \vdash B$, then $\Delta, \Gamma \vdash B$

As we can see from definition 1, the properties are preserved by the notion of a fictional implication. Informal intuition:

(Reflexivity) Suppose that $A \in \Gamma_k$, that is, A is an assertion of the fictional world k . In fact, Γ_k fictionally implies A in the story k , because A is asserted in the story k .

(Monotonicity) Suppose Γ_k fictionally implies B . If I continue the story and add more facts to it, it will still imply B . Therefore, Γ_k and A imply B .

(Cut) Suppose Δ_k fictionally implies A and Γ_l and A imply B . If we merge fictional worlds k and l , then, by monotonicity, $\Delta_k, \Gamma_l \vdash A$. By deduction theorem and monotonicity, $\Delta_k, \Gamma_l \vdash A \rightarrow B$, therefore, $\Delta_k, \Gamma_l \vdash A \rightarrow B$.

From these properties, all assertions of a fictional world k are fictionally implied by k , in virtue of reflexivity. Nevertheless, we can also imply other assertions based on the rules of LET_K^- . For example, if $A \in \Gamma_k$, and $B \in \Gamma_k$, then $\Gamma_k \vdash_{LET_K} A$ and $\Gamma_k \vdash_{LET_K} B$. Thus, by $\wedge I$, $\Gamma_k \vdash_{LET_K} A \wedge B$. This defines the scope of our proposed minimal logic for reasoning within fictional contexts. We do not propose to rule out other types of inferences and subtleties involved in reading a work of art. On the contrary, we emphasize this aspect by restricting deductive fictional implication to simple rules involving the connectives. Furthermore, we can

provide a valuation semantics for LET_K^- , as presented in Coniglio and Rodrigues (2024, p. 566):

Definition 2. Let ϑ be a function from formulas to $\{0,1\}$. ϑ is a bivaluation for LET_K^- if it satisfies that

- $v1 \quad \vartheta(A \wedge B) = 1 \text{ iff } \vartheta(A) = 1 \text{ and } \vartheta(B) = 1$
- $v2 \quad \vartheta(A \vee B) = 1 \text{ iff } \vartheta(A) = 1 \text{ or } \vartheta(B) = 1$
- $v3 \quad \vartheta(A \rightarrow B) = 1 \text{ iff } \vartheta(A) = 0 \text{ or } \vartheta(B) = 1$
- $v4 \quad \vartheta(\neg\neg A) = 1 \text{ iff } \vartheta(A) = 1$
- $v5 \quad \vartheta(\neg(A \wedge B)) = 1 \text{ iff } \vartheta(\neg A) = 1 \text{ or } \vartheta(\neg B) = 1$
- $v6 \quad \vartheta(\neg(A \vee B)) = 1 \text{ iff } \vartheta(\neg A) = 1 \text{ and } \vartheta(\neg B) = 1$
- $v7 \quad \vartheta(\neg(A \rightarrow B)) = 1 \text{ iff } \vartheta(A) = 1 \text{ and } \vartheta(\neg B) = 1$

4.2 LET_K : how to talk about fiction

LET_K extends LET_K^- and it provides a way for paracomplete and paraconsistent phenomena to coexist with classical ones. The Law of Excluded Middle and the Principle of Explosion are not valid in general; however, it's possible to recover classicality for some proposition A , if we assume that $\circ A$ holds. Truth *simpliciter* — that is, truth about non-fictional worlds — is neither paracomplete nor paraconsistent: contradictions and cases of undeterminedness in the world can be only apparent violations of classicality, due to our limitations and our cognitive biases in relation to our experience. In this article, we will be working under the hypothesis that truth is classical, that is, we are not considering dialetheism and its paracomplete counterpart, analetheism, as exposed in Beall and Ripley (2004), Armour-Garb and Priest (2005), which defend that there are real contradictions (or real undeterminedness).¹⁰ It is debatable whether true simpliciter may be classical considering fictional contradictions, as pointed in Everett (2005). However, this will not be further discussed here. In Carnielli and Rodrigues (2019, p. 2), parafinite logics are divided between an ontological interpretation (dialetheism and analetheism), a pragmatic interpretation, and an epistemic interpretation, which, in the case of LETs, is an interpretation in terms of preservation of evidence:

¹⁰ It is not the aim of this article to defend that truth *simpliciter* is classical. We only start from this point of view to contrast it with fictional truth.

The dialetheist claims that some contradictions are ontological in the sense that they are due to some ‘inner contradictory essence of reality’ [...] Another view in paraconsistency, that we call metaphysical neutrality, ‘suspends judgement’ with respect to the nature of contradictions. It is a somewhat pragmatic position: the fact that contradictions occur in several contexts of reasoning is enough to justify a non-explosive account of logical consequence, and there is no need to go into the metaphysical question about the nature of such contradictions. [...] A third position in paraconsistency, antagonistic to dialetheism, claims that no contradiction is ontological but, rather, all contradictions that occur in scientific theories, belief systems, a number of situations in informal reasoning, and even in semantic and set theoretical paradoxes — that are, strictly speaking, results about languages with certain characteristics — have epistemic character in the sense that they are related to thought and language.

In this way, we can interpret $\circ A$, the recovery of classicality, as stating that “ A is a non-fictional assertion”. It excludes fictional discourse, which is how we interpret the negation of the $\circ$ operator here. Later, metafictional truth will be characterized under this condition. Accordingly, A is treated as a classical truth: PE° and PEM° hold, because classical truth admits no gluts nor gaps. As is pointed out in Barrio (2018, p. 91), all classical rules are valid for consistent propositions.

$$PE^\circ \frac{A \quad \neg A}{B} \quad PEM^\circ \frac{\circ A}{A \vee \neg A}$$

In terms of the semantics of valuations, LET_K adds one condition to the valuation for LET_K : $\vartheta(\circ A)$ implies $\vartheta(\neg A) = 1$ iff $\vartheta(A) = 0$, as presented in Carnielli and Rodrigues (2019, 3805). For those not familiar with Logics of Formal Inconsistency (LFIs) and LETs, it may seem confusing to introduce a $\circ$ operator. How is it possible for a logic to work with two types of inferences, classical and paraconsistente (or parafinite) at the same time? The idea is simply that under the assumption of $\circ A$, A behaves classically. Otherwise, it behaves parafinitely. There is no change of meaning of the values “0” and “1” depending on whether the sentence is prefixed with $\circ$. What happens is that we can reason classically with A in that case. LET_K allows for the following possible truth-states of a proposition:

A is true simpliciter (T)	iff	$\vartheta(A) = 1, \vartheta(\neg A) = 0, \vartheta(\circ A) = 1$
A is false simpliciter (F)	iff	$\vartheta(A) = 0, \vartheta(\neg A) = 1, \vartheta(\circ A) = 1$
A is fictionally true (t)	iff	$\vartheta(A) = 1, \vartheta(\neg A) = 0, \vartheta(\circ A) = 0$
A is fictionally false (f)	iff	$\vartheta(A) = 0, \vartheta(\neg A) = 1, \vartheta(\circ A) = 0$

$$\begin{array}{ll}
A \text{ is fictionally overdetermined (o)} & \text{iff } \vartheta(A) = 1, \vartheta(\neg A) = 1, \vartheta(\circ A) = 0 \\
A \text{ is fictionally undetermined (u)} & \text{iff } \vartheta(A) = 0, \vartheta(\neg A) = 0, \vartheta(\circ A) = 0
\end{array}$$

A more complicated question is what it means for A to hold in a fictional world k . If $\vartheta(\circ A) = 1$, then A is a non-fictional assertion; thus, we already have criteria to decide whether A or $\neg A$ holds since we are dealing with (classical) truth preservation. If $\vartheta(\circ A) = 0$, then A is a fictional assertion. In this case, what does it mean for A to hold? Fictional implication is defined in an analogous way to LET_K , the difference being that in LET_K , we can consider not only fictional assertions, but also parafictional and metafictional ones.

As exposed in Coniglio and Rodrigues (2024), the valuation semantics for LET_K is sound and complete with respect to its natural deduction system.¹¹ Thus, we can define a characteristic valuation ϑ_k that is the adequate characterization of the fictional world k .

Definition 4. Let Δ_k be the set of all assertions of a fictional world k , we define the characteristic valuation of k , ϑ_k , as

$$\vartheta_k(A) = 1 \text{ iff } \Delta_k \models_{LET_K} A$$

As the definitions above illustrate, this logical system can analyze fictional truths, as it becomes clear from the definitions above, and truths about fictional worlds, that is, metafictional truths. Metafictional truths are factual in nature; the key distinction is that in such cases, $\circ A$ holds. Parafictionality appears to straddle both metafictional and fictional contexts, as it is expressed by Recanati (2018, p. 5), “An utterance containing a fictional name is explicitly parafictional when it starts with a phrase such as ‘in the fiction...’, or ‘according to the story’...”. In our framework, we simplify the role of parafictionality as follows: a parafictional statement takes the (implicit or explicit) form: ‘According to story k , A .’ This can be formalized as an assertion of A , accompanied by the negation of the consistency of A .

A is a parafictional sentence according to S iff A is of the form $B \wedge \neg \circ B$

¹¹ This logic can also be characterized in terms of NMatrices. However, this is a choice of presentation, as both semantics – NMatrices and valuations – determine the *same* logic, a fact which can be proved through a representation theorem.

For example, the sentence “In the story of Spider-Man, Peter Parker likes Mary Jane” is a parafictional sentence, because it says (1) Peter Parker likes Mary Jane, and (2) “Peter Parker likes Mary Jane” is a fictional sentence. Let A be the statement: “In the story of Spider-Man, Peter Parker likes Mary Jane”, and B : “Peter Parker likes Mary Jane”. Sentence A can be formalized as $B \wedge \neg \circ B$.

This does not mean that the sentence B is contradictory: LET_K can differentiate six situations, in which four of them are not consistent in a broader sense: fictional truth (t), fictional falsehood (f), fictional overdeterminedness (o), and fictional undeterminedness (u). In a strict sense of being coherent, the fact $\neg \circ B$ is the case does not imply that the story is incoherent: it can be the case that all determined propositions are either fictionally true or false.

In a logic of fictional discourse, however, there is one aspect of fictional truth that we must include in LET_K for it to maintain intuitions about fictional discourse. When we have two statements that are on the same level of discourse (i.e. they are both fictional or they are both factual), we want their compound statements to remain at the same level of discourse. In the same way, the negation of a fictional statement should remain fictional, and the negation of a factual statement should remain factual. For example, consider the statement “Harry Potter is a wizard” and “Voldemort is a wizard”. Both statements are fictional. Therefore, we want the statement “Harry Potter is a wizard and Voldemort is a wizard” to remain fictional. However, LET_K can’t decide, in this case, between this statement being fictionally or factually true (it does not have clauses for consistency propagation). Thus, we add the following conditions for bivaluations:

$$vC1 \quad \vartheta(\circ A) = 1 \text{ and } \vartheta(\circ B) = 1 \Rightarrow \vartheta(\circ (A \# B)) = 1, \# \in \{\wedge, \vee, \rightarrow\}$$

$$vC2 \quad \vartheta(\circ A) = 0 \text{ and } \vartheta(\circ B) = 0 \Rightarrow \vartheta(\circ (A \# B)) = 0, \# \in \{\wedge, \vee, \rightarrow\}$$

$$vC3 \quad \vartheta(\circ A) = \vartheta(\circ \neg A)$$

With these conditions, it becomes clear that fictionality and factuality is propagated. Suppose, for example, that A is a factual truth. In that case $\vartheta(A) = 1, \vartheta(\neg A) = 0, \vartheta(\circ A) = 1$. Is $\neg A$ a factual statement? By condition $vC3$, we know that $\vartheta(\circ A) = \vartheta(\circ \neg A)$, which means that $\neg A$ is a factual statement.

The extension of LET_K with the following axioms validates the conditions given above:

$$AC1 \quad (\circ A \wedge \circ B) \rightarrow \circ (A \# B), \# \in \{\rightarrow, \wedge, \vee\}$$

$$AC2 \quad \circ (A \# B) \rightarrow (\circ A \vee \circ B), \# \in \{\rightarrow, \wedge, \vee\}$$

$$AC3 \quad \circ A \leftrightarrow \circ \neg A$$

Soundness proofs for axioms (AC1)–(AC3):

- (AC1) Suppose that $\vartheta(\circ A \wedge \circ B) = 1$. In that case, $\vartheta(\circ A) = 1$ and $\vartheta(\circ B) = 1$. By condition vC1, that implies $\vartheta(\circ (A \# B)) = 1$.
- (AC2) Suppose that $\vartheta(\circ (A \# B)) = 1$. By condition vC2, by contraposition, this implies that $\vartheta(\circ A) = 1$ or $\vartheta(\circ B) = 1$, therefore, $\vartheta(\circ A \vee \circ B) = 1$.
- (AC3) Straightforward proof left as an exercise to the reader.

In relation to completeness, we will only sketch the idea here. When constructing the maximal consistent set Γ , the step required is to prove that the valuation produced by Γ is a LET_K valuation. That is, we need to prove that

$$\varphi \in \Gamma \quad \text{iff} \quad \vartheta_c(\varphi) = 1,$$

where ϑ_c is the canonical valuation defined for Γ . Therefore, we need to prove that the set Γ satisfies the conditions (vC1)–(vC3). Using axioms (AC1)–(AC3), it's straightforward to check for conditions above, as Γ is deductively closed. LETs are further explored in Coniglio and Rodrigues (2024), in which the authors expose 6-valued LETs, LET_K^+ and LET_F^+ . The issue of system choice will not be further explored here. The main point to stress is that these LETs provide the adequate logical space to distinguish between fictional truth (and falsehood), metafictional truth (and falsehood), overdeterminedness and undeterminedness.

5 Objections

This approach to fiction based on LET_K may cause some discomfort at first glance, and it seems that this approach is incompatible with other visions of fictional truth. In this section, we hope to clear up some misunderstandings and explain how our logic of fiction can be non-controversial in many aspects. The first objection goes as follows: if a fictional assertions is possibly contradictory, then what if I assert a sentence fictionally that says that my story is not

contradictory, for example? Does it mean that I am asserting something metafictional inside a fiction?

This objection will help to enlighten the extent to which our logical fictional discourse can be applied. Suppose that we have a fictional story in which I assert “The sentence p_1 is not contradictory”. What I am asserting here is that $\neg(p_1 \wedge \neg p_1)$. However, as we stipulated, every fictional sentence A is such that $\circ A$ does not hold, thus, $\vartheta(\circ \neg(p_1 \wedge \neg p_1)) = 0$. Therefore, the proposition $\neg(p_1 \wedge \neg p_1)$ is still possibly contradictory, that is, there is a valuation in which $\vartheta(p_1 \wedge \neg p_1) = 1 = \vartheta(\neg(p_1 \wedge \neg p_1)) = 1$. In other words, saying that some sentence is not contradictory is no guarantee of non-contradiction in fiction, because it is always possible to extend the story with incompatible elements.

Another aspect of LET_K that is worth exploring is the relation between mixed statements, that is, statements that mix fictional and factual statements. Consider the following example: let A : “Peter Parker is a super-hero”, and B : “Gilberto Gil is a brazilian singer”. A is a fictional truth (t), whereas B is a factual truth (T). What happens when we consider the conjunction of such statements, that is, $A \wedge B$?

Using bivaluations, we know that A is a fictional truth iff $\vartheta(A) = 1, \vartheta(\neg A) = 1, \vartheta(\circ A) = 0$. B is a factual truth iff $\vartheta(B) = 1, \vartheta(\neg B) = 1, \vartheta(\circ B) = 1$. Therefore, using the clauses for bivaluation presented by Carnielli and Rodrigues (2019, p. 3798), we can infer that $\vartheta(A \wedge B) = 1$ and $\vartheta(\neg(A \wedge B)) = 0$. What is the value of $\vartheta(\circ(A \wedge B))$? As it turns out, LET_K is still neutral with this respect: the current valuation is not sufficient to determine the value of $\circ(A \wedge B)$. That is, there is a valuation ϑ_1 that extends ϑ such that $\vartheta_1(\circ(A \wedge B)) = 1$ and there is a valuation ϑ_2 that extends ϑ such that $\vartheta_2(\circ(A \wedge B)) = 0$. In other words, LET_K does not decide whether a mixed statement is a fictional truth or a factual truth. That’s because the only condition (besides the consistency propagation clauses we introduced above) on the $\circ$ operator in the bivaluation is that

$$vC4 \quad \vartheta(\circ A) = 1 \Rightarrow \vartheta(A) = 1 \text{ iff } \vartheta(\neg A) = 0$$

Fixing this problem is easy: we just need to add consistency propagation clauses. In the case of conjunction, this would amount to adding the following condition:

$$vC5 \quad \vartheta(\circ(A \wedge B)) = 1 \Rightarrow \vartheta(\circ A) = 1 \text{ and } \vartheta(\circ B) = 1$$

Nevertheless, we do not think this is necessarily a problem. As we discussed before, our approach intends to be more neutral with respect to interpretations of fictional discourse: its purpose is to serve as a shared common ground upon which interpretations of fictional discourse can be compared. Therefore, the question of the truth-value of mixed statements is still left open, and we can extend LET_K according to the interpretation we want to support. In the present case, adding (vC5) to the clauses of bivaluation would mean that mixing fictional and factual truths results in a fictional truth. Clearly, this already takes a stance on the debate of mixed statements. On the other hand, we could add the following condition to make the mix between fictional truth and factual truth another factual truth:

$$vC6 \quad \vartheta(\circ A) = 1 \text{ or } \vartheta(\circ B) = 1 \Rightarrow \vartheta(\circ (A \wedge B)) = 1$$

Another advantage of LET_K is its ability to differentiate between types of non-classicality involved in a fictional story. Some fictional universes may be coherent in the sense of not being contradictory, that is, there is no proposition that receives the value $\circ$ in the story, despite being incomplete, that is, some propositions receive the value u . On the other hand, some fictional universes may be intentionally contradictory such as Priest's *Sylvan's Box*. In that case, the proposition "The box was really empty and occupied at the same time." is a fictional overdetermination.

Finally, a third objection is that what we are modelling when we think of LET_K as a logic of fiction is not fictional truth, but parafictional truth. In regard to truth in fiction, it can behave as the author may wish: it can reason intuitionistically, classically, etc.¹² There are two points here to be made: first, as we characterized parafictional truth, its form needs to be $A \wedge \neg \circ A$, that is, it needs to be prefixed by "According to the story...". Therefore, no assertion within the work is parafictional — let aside metalanguage statements. Furthermore, our claim is that fictional discourse has one logic, and that must be paraconsistent and paracomplete. Some inferences are always legitimate to make as part of the fictional game. For example, if the narrative asserts A and B , it is forced upon the story to accept $A \wedge B$, independently of the logic the author supposedly wants the fiction to behave. If the author wants the story to be classical, just willing to do so won't make it classical. Fiction is still an area where contradictions may rise. LET_K and its extensions that we showed above provide a good basis to think in these

¹² We thank one reviewer for making this objection.

terms: even when there is no contradiction in the story, it still fails to be classically valid, because $\circ$ does not hold in that case.

6 A 6-valued account of fictional truth

So far, we have been using bivaluations and the names $\{T, t, f, F, o, u\}$ were only labels for a three-dimensional picture of the bivaluation semantics. Notwithstanding, there are stronger LETs that can be characterized in terms of many-valued semantics. In this section, we will consider the logic LET_K^+ , a 6-valued LET. A^T is defined as $A \wedge \circ A$, and A^F is defined as $\neg A \wedge \circ A$.

Definition 5. LET_K^+ is defined by a natural deduction system with the following rules (CONIGLIO; RODRIGUES, 2024, p. 576), together with the rules of LET_K .

$\frac{}{\circ \circ A}$	$\frac{\circ \neg A}{\circ A}$	$\frac{\circ A}{\circ \neg A}$
$\frac{A^T \quad B^T}{(A \wedge B)^T}$	$\frac{A^F}{(A \wedge B)^F}$	$\frac{B^F}{(A \wedge B)^F}$
$\frac{A^T}{(A \vee B)^T}$	$\frac{B^T}{(A \vee B)^T}$	$\frac{A^F \quad B^F}{(A \vee B)^F}$
$\frac{A^F}{(A \rightarrow B)^T}$	$\frac{B^T}{(A \rightarrow B)^T}$	$\frac{A \quad B^F}{(A \rightarrow B)^F}$
$\frac{(A \wedge B)^T}{A^T}$	$\frac{(A \wedge B)^T}{B^T}$	$\frac{(A \wedge B)^F \quad \frac{[A^F]}{C} \quad \frac{[B^F]}{C}}{C}$
$\frac{(A \vee B)^T \quad \frac{[A^T]}{C} \quad \frac{[B^T]}{C}}{C}$	$\frac{(A \vee B)^F}{A^F}$	$\frac{(A \vee B)^F}{B^F}$
$\frac{(A \rightarrow B)^T \quad \frac{[A^F]}{C} \quad \frac{[B^T]}{C}}{C}$	$\frac{(A \rightarrow B)^F}{A}$	$\frac{(A \rightarrow B)^F}{B^F}$

Even though LET_K^+ can be characterized with a bivaluation semantics, it can also be characterized with a 6-valued semantics.

Definition 6. Let $M_{LET_K^+} = (A, D)$, $A = \{T, t, o, u, f, F\}$, $D = \{T, t, o\}$. The truth tables of $M_{LET_K^+}$ are the following:

	$\neg$	$\circ$
T	F	T
t	f	F
o	o	F
u	u	F
f	t	F
F	T	T

$\wedge$	T	t	o	u	f	F
T	T	t	o	u	f	F
t	t	t	o	u	f	F
o	o	o	o	f	f	F
u	u	u	f	u	f	F
f	f	f	f	f	f	F
F	F	F	F	F	F	F

$\vee$	T	t	o	u	f	F
T	T	T	T	T	T	T
t	T	t	t	t	t	t
o	T	t	o	t	o	o
u	T	t	t	u	u	u
f	T	t	o	u	f	f
F	T	t	o	u	f	F

$\rightarrow$	T	t	o	u	f	F
T	T	t	o	u	f	F
t	T	t	o	u	f	F
o	T	t	o	u	f	F
u	T	t	t	t	t	t
f	T	t	t	t	t	t
F	T	T	T	T	T	T

Talking about fictional and metafictional statements becomes more intuitive in this semantics: statements that receive the value T are factual truths, whereas those that receive the value F are factual falsehoods. The value t is a fictional truth and the value f a fictional falsehood, whereas o and u are fictional overdeterminedness and undeterminedness, respectively.

For all formulae φ such that φ is a statement related to a fictional world k , φ is a metafictional statement of k iff $\vartheta(\varphi) \in \{T, F\}$. Otherwise, φ is a fictional statement. This feature also underlines Recanati (2018, p. 1), which has the idea that cases of fictional, metafictional and parafictional discourse depend on the utterances of those sentences, that is, if they are in a fictional, or metafictional, context. A sentence φ can receive a fictional or a non-fictional value, without altering its logical form.

LET_K^+ , however, possesses additional features. The advantage is that when considering non-mixed contexts (i.e., only fictional or only metafictional statements), the compound statements will always remain closed under the same context (except if you consider the factuality operator $\circ$). On the other hand, LET_K^+ takes a stance on mixed statements: fictionality does not spread in all cases. For example, if A is a fictional truth and B is a factual truth, $A \vee B$ results in a factual truth.

7 Other logical approaches to fictional discourse

There are other ways to represent fictional discourse logically available in the literature. One closely related approach to ours presented in Peron and Antunes (2022) is a four-valued approach that compares fictional truth (t), fictional falsehood (f), factual truth (T), and factual falsehood (F). This work also introduces a fictionality operator $!$ that is such that $!\varphi$ means that φ is a factual statement. This operator, however, behaves deterministically.

Consider the proposition “ φ is a factual statement”. If we are in the 4-valued paradigm by Peron and Antunes, then this statement is always factual, because their connective of factuality ! only allows for factual values. This becomes a problem when we consider this statement inside a work of fiction, because this means that we cannot fictionalize our use of “it is a fact that..”, using the same operator. The $\circ$ operator presented for LET_K , however, has a non-deterministic behavior: it allows for the statement that “ φ is a fact”, φ a factually true (false) statement, to be fictionally true (false), for example. In that case, a fictional character that asserts a factual truth would be asserting a fictional truth in a fictional world where facts of the world are assumed to hold.

Furthermore, when presenting the truth-tables for the propositional connectives, they must introduce two operator of conjunction, $\wedge_1$ and $\wedge_2$, each one representing one interpretation of mixed statements of fictionality and factuality. Our non-deterministic approach can represent this neutrality by encompassing both possibilities within a single NMatrix, which is a suggestion they also recognize in Peron and Antunes (2022, p. 606).

Nevertheless, Peron and Antunes go one step further in relation to a first-order logic of fiction, in which quantified statements are also dealt with. It remains a non-trivial problem to answer how to combine the interpretation of LET_K as a logic of fiction and truth together with quantifiers, which pose a challenge by themselves. It would also be desirable to do this for extensions of LET_K up to the 6-valued LET_K^+ . Although lacking some desirable features we found on LET_K that is brought up by non-determinism, LET_K^+ presents an intuitive way to think of fictional discourse encompassing both paraconsistent and paracomplete phenomena.

Classical approaches such as Lewis (1978), in our view, suffer from an expressive limitation: they cannot distinguish cases of fictional overdetermination and undeterminedness. Other approaches try to overcome this limitation by incorporating non-classical aspects to classical approaches such as Badura and Berto (2019), which deal with inconsistencies in fiction. Again, our logic fares better in terms of expressiveness, because we can syntactically represent fictional discourse in comparison to factual discourse, and we can represent inconsistent and incomplete phenomena.

8 Conclusions and further remarks

In this paper, we discuss and compare different uses of fictional discourse, and its relation to factual discourse. Using LET_K and its extensions, we have enough expressive power to distinguish between factual and fictional discourse, along with fictional undeterminedness and overdeterminedness. Further exploration of this topic would involve considering first-order extensions of LET_K as in Rodrigues and Antunes (2022), which we present in the propositional fragment. A major discussion of the logic of fiction, however, concentrates on the topic of singular terms in fiction, such as “Sherlock Holmes”, “Spider Man”. These cases can only be analyzed with a first-order extension of LET_K . Nevertheless, the 6-valued NMatrix semantics presented for LET_K^+ poses a good ground to start the discussions on fictional sentences. The question of whether fictional terms should have references on the domain of objects is left for further work. Another good perspective to fictional discourse would be to think of LETs in terms of Kripke semantics as in Antunes et al. (2020). Future research of this topic also includes formalizing the notion of parafictional discourse, which has been investigated briefly here; investigating concrete cases of contradictions and undeterminedness in fiction; investigating cases of art works that mix factual and fictional contexts.

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